Solve $y'' = (y')^2 \tan y$. **SCORE: _____ / 9 PTS**

$u = \frac{dy}{dx}$

$u\frac{du}{dy} = \frac{d^2y}{dx^2}$

EACH UNDERLINED ITEM WORTH 1 POINT EXCEPT THOSE OTHERWISE INDICATED

$u\frac{du}{dy} = u^2 \tan y$ (2)

$\int \frac{1}{u}\,du = \int \tan y\,dy$

$\ln|u| = \ln|\sec y| + C$ (2)

$u = C\sec y$

$\frac{dy}{dx} = C\sec y$

$\int \cos y\,dy = \int C\,dx$

$\sin y = Cx + D$

$y = \sin^{-1}(Cx+D)$

COULD $u = 0$ BE A SOLUTION?

$u = 0 \longrightarrow \frac{dy}{dx} = 0$

$y = K$

$y' = 0$

$y'' = 0$ EQUAL

$(y')^2 \tan y = 0$

IF $y \neq \frac{\pi}{2} + n\pi$

SO $y = K \neq \frac{\pi}{2} + n\pi$

BONUS (1) POINT

Use elimination (as shown in lecture) to solve the system **SCORE: _____ / 21 PTS**

$$(2D+3)(x)-(D+3)(y) = 5$$
$$(D+1)(x) - (2D+5)(y) = 7+4t$$

APPLY
$D+1$ TO 1^{ST} EQ'N
$2D+3$ TO 2^{ND} EQ'N

$$(D+1)(2D+3)(x)-(D+1)(D+3)(y) = 0+5 = 5$$
$$-\ (2D+3)(D+1)(x)-(2D+3)(2D+5)(y) = 2(4)+3(7+4t) = 29+12t$$

$$(3D^2+12D+12)(y) = -24-12t$$
$$(D+2)^2(y) = -8-4t$$
$$y_h = Ae^{-2t}+Bte^{-2t}$$
$$y_p = K_1t+K_2$$
$$y_p' = K_1$$
$$y_p'' = 0$$
$$y_p''+4y_p'+4y_p = 0+4K_1+4K_1t+4K_2 = -8-4t$$
$$4K_1 = -4 \rightarrow K_1 = -1$$
$$4K_1+4K_2 = -8 \rightarrow K_2 = -1$$
$$y = -t-1+Ae^{-2t}+Bte^{-2t}$$

APPLY
$2D+5$ TO 1^{ST} EQ'N
$D+3$ TO 2^{ND} EQ'N

$$(2D+5)(2D+3)(x)-(2D+5)(D+3)(y) = 2(0)+5(5) = 25$$
$$-\ (D+3)(D+1)(x)-(D+3)(2D+5)(y) = 4+3(7+4t) = 25+12t$$

$$(3D^2+12D+12)(x) = -12t$$
$$(D+2)^2(x) = -4t$$
$$x_h = Ce^{-2t}+Dte^{-2t}$$
$$x_p = K_3t+K_4$$
$$x_p' = K_3$$
$$x_p'' = 0$$
$$x_p''+4x_p'+4x_p = 0+4K_3+4K_3t+4K_4 = -4t$$
$$4K_3 = -4 \rightarrow K_3 = -1$$
$$4K_3+4K_4 = 0 \rightarrow K_4 = 1$$
$$x = -t+1+Ce^{-2t}+Dte^{-2t}$$

$$(2D+3)(x)-(D+3)(y)$$

$$= \boxed{\begin{aligned}&2(\quad -1-2Ce^{-2t}+De^{-2t}-2Dte^{-2t})\\ &+3(-t+1+Ce^{-2t}+Dte^{-2t})\\ &-(\quad -1-2Ae^{-2t}+Be^{-2t}-2Bte^{-2t})\\ &-3(-t-1+Ae^{-2t}+Bte^{-2t})\end{aligned}}$$

$$= \underline{\cancel{5}+(-A-B-C+2D)e^{-2t}+(-B-D)te^{-2t}=\cancel{5}}$$

$$-B-D=0 \longrightarrow \underline{D=-B}$$

$$-A-B-C+2D=0 \longrightarrow -A-3B-C=0$$

$$\underline{C=-A-3B}$$

$$\boxed{\begin{aligned}x &= -t+1-(A+3B)e^{-2t}-Bte^{-2t}\\ y &= -t-1+Ae^{-2t}+Bte^{-2t}\end{aligned}}$$

ALTERNATELY

$$-B-D=0 \longrightarrow B=-D$$

$$-A-B-C+2D=0 \longrightarrow -A-C+3D=0$$

$$A=-C+3D$$

$$x=-t+1+Ce^{-2t}+Dte^{-2t}$$

$$y=-t-1-(C-3D)e^{-2t}-Dte^{-2t}$$

ALTERNATELY

$$
\begin{aligned}
&(D+1)(x)-(2D+5)(y)\\
&= (\quad -1-2Ce^{-2t}+De^{-2t}-2Dte^{-2t})\\
&\quad +(-t+1+Ce^{-2t}+Dte^{-2t})\\
&\quad -2(\quad -1-2Ae^{-2t}+Be^{-2t}-2Bte^{-2t})\\
&\quad -5(-t-1+Ae^{-2t}+Bte^{-2t})\\
&= \cancel{4t}+\cancel{7}+(-A-2B-C+D)e^{-2t}+(-B-D)te^{-2t}=\cancel{7}+\cancel{4t}
\end{aligned}
$$

$$-B-D=0 \longrightarrow D=-B$$

$$-A-2B-C+D=0 \longrightarrow -A-3B-C=0$$

$$C=-A-3B$$

$$x=-t+1-(A+3B)e^{-2t}-Bte^{-2t}$$

$$y=-t-1+Ae^{-2t}+Bte^{-2t}$$

ALTERNATELY

$$-B-D=0 \longrightarrow B=-D$$

$$-A-2B-C+D=0 \longrightarrow -A-C+3D=0$$

$$A=-C+3D$$

$$x=-t+1+Ce^{-2t}+Dte^{-2t}$$

$$y=-t-1-(C-3D)e^{-2t}-Dte^{-2t}$$